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EFFECT OF SMALL INITIAL DEFLECTION ON EFFECTIVE WIDTH OF WIDE SHIP PLATING - NUMERICAL RESULTS

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Effect of Small Initial Deflection on Effective Width of Wide Ship Plating--Numerical Results

by

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NOMENCLATURE

Symbols	Definitions
a	Plate length in the x-direction
b	Plate breadth in the y-direction
b_e	Effective width
K	Buckling factor
t	Plate thickness
w	Net plate deflection
w_o	Initial deflection
x, y, z	Coordinate axes
D	Flexural rigidity of plating = $Et^3/12(1 - \nu^2)$
E	Young's modulus
F	Airy's stress function
β	Aspect ratio, b/a
ν	Poisson's ratio
σ_o	Average compressive load in the x-direction
σ_{cr}	Buckling stress
σ_{bx}, σ_{by}	Bending stresses
σ_{mx}, σ_{my}	Membrane stresses
λ	Nondimensional load factor
λ_{cr}	Nondimensional buckling load
ψ	Excess of buckling load
u, v, w	Displacements of a point in the x, y and z directions, respectively
$\epsilon_x, \epsilon_y, \gamma_{xy}$	Median-fiber strains

Symbols	Definitions
$\sigma_x, \sigma_y, \tau_{xy}$	Median-fiber stresses
a_{mn}	Given initial deflection coefficients
b_{mn}	Unknown coefficients concerning net deflection
ρ	Ratio of effective width to actual width or effectiveness of plates
P_x, P_y	Total loads in the x and y directions, respectively.
p	Lateral load

I. INTRODUCTION

Naval architects are well aware of the weakening effect of deflections in plating under compression. An analysis of the effectiveness of the plate, defined as the ratio of the contraction or extension of a deflected plate to that of a perfectly plane plate under the same in-plane edge forces, has been investigated by Murray [8] and by Horne [9]. Murray's conclusion is that the deflections noted in the plating of merchant ships do not, in general, have an important effect on the effectiveness of the structure unless the plating is very thin. For plating in compression, he suggests that the initial deflection should not exceed 0.3 times the thickness of the plating if great loss in efficiency is to be avoided.

An investigation for plating in aircraft structure was undertaken by Hu, Lundquist, and Batdorf [10]. Their conclusions for plates in compression are:

(a) The effects of initial deflections upon the buckle growth and effective width of simply supported square plates are most marked near the theoretical flat-plate critical stress. At stresses well above or below the critical stress, the behavior of a plate with an initial deflection is very much the same as that of a plate that is initially perfectly flat.

(b) The effective width for load-carrying capacity of an initially deflected plate is at all values of stress less than that of an initially flat plate.

The way in which the net center deflection increases with the average edge stress σ_o is shown in Fig. 151 of reference [7]. It shows that when

---

Note: [ ] indicates reference number under References.

a plate with an initial deflection is compressed, the deflections grow slowly at first and then rapidly as the critical load is approached. In the case of the perfectly flat plate, of course, there is no deflection at all until the critical load is reached. As seen from Fig. 151, the rate of increase of net center deflection approaches the deflection for the perfectly flat plate at values not far above the critical load. The above explanations also describe Fig. 3-a of this paper.

In this present paper the author approaches the same problem by large deflection theory for three different aspect ratios using one term in the longitudinal direction and fifteen terms in the transverse direction. Its theoretical part was developed by Marguerre [3] for the plate with initial deflections as far as the author knows. Levy [2] solved the fundamental equations exactly for simply supported plate, and Yamaki [1] further developed Marguerre's formulas for various boundary conditions. Recently Dr. Schultz [5] performed some experiments of post-buckling behavior of wide ship plate considering the initial deflections without the theoretical calculations. He did the theoretical calculations only for the flat-plate case.

Upon the suggestion of Dr. Schultz, the author has investigated the effect of small initial deflections upon the effective width of wide ship plating based on Yamaki's [1] equations using an IBM 7090.

The present investigation is limited to simply supported edges with aspect ratio β of 3.5, 5.0 and 6.5. The author also assumed that the initial deflections were caused by uniformly distributed lateral loads. The author employed the computer programming developed by Shao [6] for the

calculations of initial deflection coefficients for three different aspect ratios for steel plate, assuming that the Poisson ratio is 0.3.

II. YAMAKI'S SOLUTION OF MARGUERRE'S FUNDAMENTAL EQUATIONS

In the following, a short summary of Yamaki's solutions of Marguerre's fundamental differential equations for the finite deflection of initially deflected thin plates will be given to the extent necessary for comprehension of the procedure to be used in obtaining numerical results.

1. Marguerre's Fundamental Equations

$$\frac{1}{E}\nabla^4 F = \frac{\partial^4 F}{\partial x^4} + 2\,\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = \left[\left(\frac{\partial^2 (w_o + w)}{\partial x \partial y}\right)^2 - \frac{\partial^2 (w_o + w)}{\partial x^2}\cdot\frac{\partial^2 (w_o + w)}{\partial y^2}\right] - \left[\left(\frac{\partial^2 w_o}{\partial x \partial y}\right)^2 - \frac{\partial^2 w_o}{\partial x^2}\,\frac{\partial^2 w_o}{\partial y^2}\right]$$

$$= \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} + 2\,\frac{\partial^2 w_o}{\partial x \partial y}\,\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w_o}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w_o}{\partial y^2}\,\frac{\partial^2 w}{\partial x^2}\,; \qquad (1)$$

$$\nabla^4 w = \frac{t}{D}\left[\frac{\partial^2 F}{\partial y^2}\,\frac{\partial^2 (w + w_o)}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\,\frac{\partial^2 (w + w_o)}{\partial y^2} - 2\,\frac{\partial^2 F}{\partial x \partial y}\,\frac{\partial^2 (w + w_o)}{\partial x \partial y}\right]. \qquad (2)$$

F is Airy's stress function, defined by

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}\,, \qquad \sigma_y = \frac{\partial^2 F}{\partial x^2}\,, \qquad \tau_{xy} = -\,\frac{\partial^2 F}{\partial x \partial y}\,. \qquad (3)$$

2. General Solution of Marguerre's Fundamental Equations

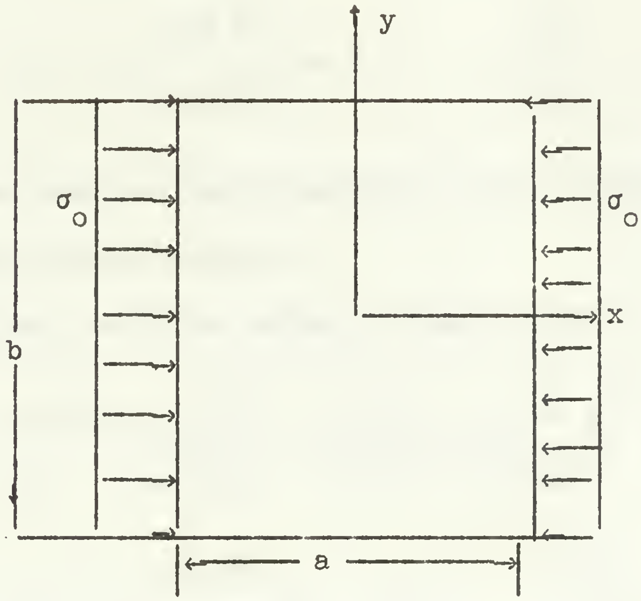


Fig. 1-a. Rectangular plate under edge compression.

We consider a plate in edge compression as shown in Fig. 1-a, simply supported at all edges, having the aspect ratios

$$\beta = b/a \ . \tag{4}$$

a. Boundary Conditions

We will state the boundary conditions for the problems here treated, Fig. 1-a. They consist of the loading and supporting conditions along the edges.

(1) Loading Conditions

Concerning the loading conditions along the side edges, $x = \pm b/2$,

$$[v]_{x} = \pm \frac{b}{2} = \int_{0}^{\pm b/2} \left[\frac{1}{E}\left(\frac{\partial^2 F}{\partial x^2} - \nu \frac{\partial^2 F}{\partial y^2} \right) - \frac{1}{2}\left(\frac{\partial w}{\partial y} \right)^2 - \frac{\partial w_o}{\partial y}\frac{\partial w}{\partial y} \right] dy = \text{const},$$

$$P_y = t\int_{-a/2}^{a/2} \frac{\partial^2 F}{\partial x^2}\, dx = 0\,, \qquad \frac{\partial^2 F}{\partial x \partial y} = 0\,. \tag{5}$$

In words, the edges are kept straight by a distribution of normal stresses, the resultant of which is zero.

The loading conditions along the loaded edges x = ±a/2 become

$$[u]_{x=\pm a/2} = \int_{0}^{\pm a/2} \left[\frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \nu \frac{\partial^2 F}{\partial x^2} \right) - \frac{1}{2}\left(\frac{\partial F}{\partial x} \right)^2 - \frac{\partial w_o}{\partial x}\frac{\partial w}{\partial x} \right] dx = \text{const},$$

$$P_x = t\int_{-b/2}^{b/2} \frac{\partial^2 F}{\partial y^2}\, dy = -\sigma_o bt\,, \qquad \frac{\partial^2 F}{\partial x \partial y} = 0\,, \tag{6}$$

where P_x is the total loads in x-direction and σ_o is the average compressive stress in the x-direction.

(2) Supporting Conditions

We will treat one case only--all edges simply supported. This condition can be expressed as follows:

$$w = 0, \qquad m_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) = 0 \text{ along } x = \pm a/2\,; \tag{7}$$

$$w = 0, \qquad m_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) = 0 \text{ along } y = \pm b/2\,. \tag{8}$$

Further, the initial deflection w_o is assumed to satisfy the same

conditions as w.

b. Determination of the Functions F and w

Now we will determine the functions F and w which satisfy the fundamental equations, together with the given boundary conditions.

The initial and additional deflections, which satisfy the boundary conditions may be expressed as

$$w_o = t \sum_m \sum_n a_{mn} \cos(m\pi x/a)\cos(n\pi y/b) \; ; \qquad (9)$$

$$w = t \sum_m \sum_n b_{mn} \cos(m\pi x/a)\cos(n\pi y/b) \; , \qquad (10)$$

where $m = n = 1, 3, 5, \cdots$.

(1) Stress Function F

In Eqs. (9) and (10), a_{mn} and b_{mn} represent the prescribed and undetermined deflection coefficients, respectively. Substituting these expressions in Eq. (1), the results can be generally expressed in the following form:

$$\nabla^4 F = \frac{\pi^4 E t^2}{a^2 b^2} \sum_{p=0,1}^{\infty} \sum_{q=0,1}^{\infty} C_{pq} \cos \frac{2p\pi}{a} x \cos \frac{2q\pi}{b} y \; , \qquad (11)$$

where C_{pq} are the quadratic functions of a_{mn} and b_{mn}.

A particular solution F_1 of Eq. (11) is given as

$$F_1 = E t^2 \sum_{p=0,1}^{\infty} \sum_{q=0,1}^{\infty} \varphi_{pq} \cos \frac{2p\pi}{a} x \cos \frac{2q\pi}{b} y \; , \qquad (12)$$

where

$$\varphi_{pq} = \frac{\beta^2}{16(\beta^2 p^2 + q^2)^2} C_{pq} \ , \ \beta = b/a \ . \tag{13}$$

Letting

$$F = -\frac{1}{2}\sigma_o y^2 + F_1 \ , \tag{14}$$

it can be verified that F satisfies Eqs. (1), (5) and (6); hence F is the stress function corresponding to this particular case.

Thus the stress function F may be generally expressed as

$$F = -\frac{1}{2}\sigma_o y^2 + Et^2 \sum_{p=0,1}^{\infty} \sum_{q=0,1}^{\infty} \varphi_{pq} \cos\frac{2p\pi}{a} x \cos\frac{2q\pi}{b} y \ . \tag{15}$$

It will be noted that φ_{pq} are, as C_{pq}, quadratic functions of a_{mn} and b_{mn}.

The determination of the coefficients C_{pq} is carried out by substituting Eqs. (9), (10) and (15) into both sides of Eq. (1) and by matching the series on both sides.

(a) Stress Function Coefficients. The author derived the general equations of φ_{pq} by a standard method, as follows. Substituting Eqs. (9) and (10) into (1), we get

$$\begin{aligned} \nabla^4 F = \frac{\pi^4 Et^2}{a^2 b^2} \sum_m \sum_{m'} \sum_n \sum_{n'} \frac{1}{4} \Big\{ & [mnm'n' b_{m'n'}(2a_{mn} + b_{mn}) - b_{m'n'}(b_{mn} m^2 m'^2 \\ & + a_{mn} m^2 n'^2 + a_{mn} m'^2 n^2)] \cos\frac{(m - m')}{a}\pi x \cos\frac{(n - n')}{b}\pi y \\ & - [mnm'n' b_{m'n'}(2a_{mn} + b_{mn}) + b_{m'n'}(b_{mn} m^2 n'^2 + a_{mn} m^2 n'^2 + a_{mn} m'^2 n^2)] \\ & \cdot \cos\frac{(m - m')}{a}\pi x \cos\frac{(n + n')}{b}\pi y - [mnm'n' b_{m'n'}(2a_{mn} + b_{mn}) \end{aligned}$$

(Equation continued on next page)

$$+ b_{m'n'}(b_{mn}m^2n'^2 + a_{mn}m^2n'^2 + a_{mn}m'^2n^2)]\cos\frac{(m + m')}{a}\pi x\cos\frac{(n - n')}{b}\pi y$$

$$+ [mnm'n'b_{m'n'}(2a_{mn} + b_{mn}) - b_{m'n'}(b_{mn}m^2n'^2 + a_{mn}m^2n'^2 + a_{mn}m'^2n^2)]$$

$$\cdot\cos\frac{(m + m')}{a}\pi x\cos\frac{(n + n')}{b}\pi y\Bigg\} . \quad (16)$$

This result can be generally expressed in the form of Eq. (11):

$$\nabla^4 F = \frac{\pi^4 Et^2}{a^2b^2}\sum_{p=0,1}^{\infty}\sum_{q=0,1}^{\infty} C_{pq}\cos\frac{2p\pi}{a}x\cos\frac{2q\pi}{b}y . \quad (11)$$

Equating Eqs. (16) and (11), we can see that Eq. (16) consists of four parts:

$$\text{when}\begin{cases} 2p = |m - m'| \\ 2q = |n - n'| \end{cases}\quad\text{and}\quad\begin{cases} 2p = m + m' \\ 2q = n + n' \end{cases}$$

$$C_{pq} = \frac{1}{4}[mnm'n'b_{m'n'}(2a_{mn} + b_{mn}) - b_{m'n'}(b_{mn}m^2n'^2 + a_{mn}m^2n'^2 + a_{mn}m'^2n^2)] ; \quad (17)$$

$$\text{when}\begin{cases} 2p = |m - m'| \\ 2q = n + n' \end{cases}\quad\text{and}\quad\begin{cases} 2p = m + m' \\ 2q = |n - n'| \end{cases}$$

$$C_{pq} = -\frac{1}{4}b_{m'n'}[mnm'n'b_{m'n'}(2a_{mn} + b_{mn}) + b_{m'n'}(b_{mn}m^2m'^2 + a_{mn}m^2n'^2 + a_{mn}m'^2n^2)] . \quad (18)$$

Substituting Eqs. (17) and (18) into (13), stress-function coefficients φ_{pq} become as follows:

$$\text{when} \begin{cases} 2p = |m - m'| \\ 2q = |n - n'| \end{cases} \quad \text{and} \begin{cases} 2p = m + m' \\ 2q = n + n' \end{cases}$$

$$\varphi_{pq} = \frac{\beta^2}{16(\beta^2 p + q^2)^2} \cdot \frac{1}{4} [mnm'n' b_{m'n'}(2a_{mn} + b_{mn}) - b_{m'n'}(b_{mn} m^2 n'^2 + a_{mn} m^2 n'^2 + a_{mn} m'^2 n^2)] \; ; \tag{19}$$

and

$$\text{when} \begin{cases} 2p = |m - m'| \\ 2q = n + n' \end{cases} \quad \text{and} \begin{cases} 2p = |m + m'| \\ 2q = |n - n'| \end{cases}$$

$$\varphi_{pq} = - \frac{\beta^2}{16(\beta^2 p + q^2)^2} \cdot \frac{1}{4} [mnm'n' b_{m'n'}(2a_{mn} + b_{mn}) + b_{m'n'}(b_{mn} m^2 n'^2 + a_{mn} m^2 n'^2 + a_{mn} m'^2 n^2)] \; . \tag{20}$$

(2) Deflection Function

For the determination of the unknown coefficients b_{mn} contained in the expressions of w and F, Yamaki [1] applied the Galerkin method to the remaining Eq. (2). This leads to the requirement that the following conditions be satisfied by each function $\cos(2p\pi x/a)\cos(2q\pi y/b)$, in terms of which the deflection w is represented.

$$\int_0^{a/2} \int_0^{b/2} \left\{ \nabla^4 w - \frac{t}{D} \left[\frac{\partial^2 F}{\partial y^2} \frac{\partial^2 (w + w_o)}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 (w + w_o)}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 (w + w_o)}{\partial x \partial y} \right] \right\} \cos \frac{2p\pi x}{a} \cos \frac{2q\pi y}{b} \, dx \, dy = 0 \; . \tag{21}$$

Substituting the expressions for w, w_o and F obtained above and integrating,

Eq. (21) yields the following general equations:

$$4r^2\lambda(a_{rs} + b_{rs}) - [3(1 - \nu^2)\beta^2]^{-1}(\beta^2 r^2 + s^2)^2 brs + \sum_m \sum_n (a_{mn} + b_{mn})$$

$$\cdot\ [(ms - nr)^2(\varphi_{(m-r)/2,(n-s)/2} + \varphi_{(m-r)/2,(s-n)/2} + \varphi_{(r-m)/2,(n-s)/2}$$

$$+ \varphi_{(r-m)/2,(s-n)/2} + \varphi_{(m+r)/2,(n+s)/2}) + (ms + nr)^2(\varphi_{(m+r)/2,(n-s)/2}$$

$$+ \varphi_{(m+r)/2,(s-n)/2} + \varphi_{(m-r)/2,(n+s)/2} + \varphi_{(r-m)/2,(n+s)/2})] = 0 ,$$

$$(m = n = r = s = 1, 3, 5, \cdot \cdot \cdot) . \qquad (22)$$

In this equation, λ is the nondimensional load factor defined as

$$\lambda = \sigma_o b^2/\pi^2 E t^2 . \qquad (23)$$

It will be noted that Eq. (22) gives a set of simultaneous equations involving cubic products of b_{mn}. Solving these equations for each prescribed value of λ, the corresponding deflection and stress functions can be determined; thus the problem is solved.

The critical load of the wide plate is given by Dr. Schultz [5] as

$$\sigma_{cr} = \frac{\pi^2 E}{12(1 - \nu^2)}\left(\frac{t}{a}\right)^2 K ; \qquad K = (1 + 1/\beta^2)^2 , \qquad (24)$$

and in nondimensional form,

$$\lambda_{cr} = K\beta^2/12(1 - \nu^2) . \qquad (25)$$

III. NUMERICAL SOLUTIONS

The theory outlined above will be applied to investigate the effect of small initial deflection on the effective width of wide ship plates having the aspect ratios of

$$\beta = 3.5,\ 5.0,\ 6.5\ .$$

The initial and additional deflection equations (9) and (10) will be limited to

$$m = 1\ ; \qquad n = 1,\ 3,\ 5,\ \cdot\ \cdot\ \cdot\ ,\ 15\ ,$$

assuming that the curvature in the x-direction is expressed by the one-term cosine. In [5] the validity of this assumption was checked by considering the case $m = n = 1, 3, 5$. Poisson's ratio is taken as $\nu = 0.3$.

1. Stress-Function Coefficients φ_{pq}

Considering $m = m' = 1$ and $n = n' = 1, 3, \cdot\cdot\cdot , 15$ only, we will have φ_{pq} index combinations as listed in Tables 1 and 2. φ_{pq} corresponding to these combinations have been developed by the author and they are tabulated in Table 3. We have 30 φ_{pq}'s.

2. Additional Deflection Coefficients, b_{mn}

Substituting the above results of φ_{pq} into the general equation (22), a set of cubic equations is obtained, which has been solved for several β values by assuming values for $b_{1,1}$ and determining the other b_{1n} values

and corresponding λ values by successive approximation.

The iteration process was carried out with an IBM-7090 electronic computer. The original computer programming for the initially flat-plate case was undertaken by Jan. The results are listed in Tables 7 through 12. Even though the computations were done for 7 different initial deflection conditions, the author limited his listing here to two cases only for each different aspect ratio. The net center deflections for three different aspect ratios for a flat plate and small initially deflected plates are shown in Fig. 3-a. The drawings have been limited to two cases for each aspect ratio.

3. Initial Deflection Coefficients, a_{mn}

We will assume that the initial deflections were caused by a small uniformly distributed lateral load. In [2] the solutions for such a case, considering one term in one direction and 6 terms in the other direction, have been investigated. In [6], Shao further developed the solutions for one term in one direction and 15 terms in the other direction and developed the IBM-7090 programming for such a case in order to get the deflection coefficients. In essence, all assumptions are the same as this case, except loading conditions. Therefore, the author made use of his computer programming in calculating various initial deflection coefficients, assuming Poisson's ratio as 0.3 instead of 0.316. The results are listed in Tables 4 through 6, and they are shown in Figs. 1 through 3.

4. Membrane Stresses

In deriving the membrane stress formula, the author followed Dr. Schultz's method. The membrane stress is given by

$$\sigma_{mx} = \partial^2 F/\partial y^2 \ , \tag{3}$$

where the stress function F is given by

$$F = -\frac{1}{2}\sigma_o y^2 + Et^2 \sum_p \sum_q \varphi_{pq} \cos\frac{2\pi p}{a} x \cos\frac{2\pi q}{b} y \ , \quad p = q = 0, 1, 2, \cdot \cdot \cdot, \tag{15}$$

where

$$\varphi_{pq} = C_{pq}\beta^2/16(\beta^2 p^2 + q^2)^2 \ . \tag{13}$$

Substituting Eq. (15) into (3), we get

$$\sigma_{mx} = \frac{\partial^2 F}{\partial y^2} = -\sigma_o - Et^2 \sum_p \sum_q \varphi_{pq} \times \frac{4\pi^2}{b^2} q^2 \cos\frac{2\pi p}{a} x \cos\frac{2\pi q}{b} y$$

$$= -\sigma_o - \frac{\pi^2 Et^2}{b^2} 4 \sum_p \sum_q \varphi_{pq} q^2 \cos\frac{2\pi p}{a} x \cos\frac{2\pi q}{b} y \ . \tag{26}$$

Because $\lambda = \sigma_o b^2/\pi^2 Et^2$,

$$\pi^2 Et^2/b^2 = \sigma_o/\lambda \ . \tag{27}$$

Substituting (27) into (26) and rearranging them, we get

$$\frac{\sigma_{mx}}{\sigma_o}\lambda = -\lambda - 4\sum_p\sum_q \varphi_{pq}q^2\cos\frac{2\pi p}{a}x\cos\frac{2\pi q}{b}y\ . \tag{28}$$

But the critical load of the wide plate is

$$\sigma_{cr} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{a}\right)^2 K\ , \tag{24}$$

where $K = (1 + 1/\beta^2)^2$. Combining Eq. (27) with (24), we get

$$\frac{\lambda}{\sigma_o} = \frac{1}{\sigma_{cr}} \cdot \frac{K}{12(1-\nu^2)}\left(\frac{b^2}{a^2}\right) = \frac{1}{\sigma_{cr}} \cdot \frac{K\beta^2}{12(1-\nu^2)}\ . \tag{29}$$

Substituting (29) into (28), we get

$$\boxed{\frac{\sigma_{mx}}{\sigma_{cr}}\ \frac{K\beta^2}{12(1-\nu^2)} = -\lambda - 4\sum_p\sum_q \varphi_{pq}q^2\cos\frac{2\pi p}{a}x\cos\frac{2\pi q}{b}y\ ,} \tag{30}$$

where $p = 0, 1$, $q = 0, 1, 2, \cdot\cdot\cdot, 15$.

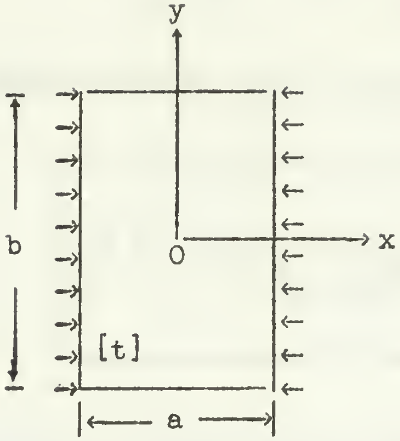
5. Effective Width

In deriving the effective-width formula, the author followed Dr. Schultz's method, instead of Yamaki's method. The effective width of a buckling plate is defined as

$$\frac{b_e}{b} = \frac{\text{average stress}}{\text{maximum stress}}\ , \tag{31}$$

or

$$\sigma_{mxe} b_e = \sigma_o b \ , \tag{32}$$



where σ_o is the compressive load, uniformly distributed over the breadth b, and σ_{mxe} is the edge membrane stress. Because it is usual practice to consider membrane stress only in effective-width computations, the membrane stress at the edge can be considered as the maximum membrane stress.

The maximum membrane stresses are found by substituting $y = \pm b/2$ into Eq. (30). Then Eq. (30) becomes

$$\frac{\sigma_{mxe}}{\sigma_{cr}} \frac{K\beta^2}{12(1 - \nu^2)} = -\lambda - 4 \sum_q \varphi_{0q} q^2 (-1)^q - 4 \sum_q \varphi_{1q} q^2 (-1)^q \cos(2\pi x/a) \ . \tag{33}$$

From Eq. (32),

$$\sigma_o/\sigma_{mxe} = b_e/b \ . \tag{34}$$

But

$$\lambda_{cr} = \sigma_{cr} b^2/\pi^2 E t^2 \ . \tag{35}$$

Substituting Eq. (24) into (35), we get

$$\lambda_{cr} = \sigma_{cr} b^2/\pi^2 E t^2 = K\beta^2/12(1 - \nu^2) \ . \tag{25}$$

Substituting Eqs. (27), (34) and (25) into the left-hand side of Eq. (33), we get

$$\frac{\sigma_{mxe}}{\sigma_{cr}} \frac{K\beta^2}{12(1 - \nu^2)} = \frac{\sigma_{mxe}}{\sigma_{cr}} \left(\frac{\sigma_{cr} b^2}{\pi^2 E t^2} \right) = \frac{\sigma_{mxe}}{\sigma_o} \lambda = \frac{b}{b_e} \lambda \ . \tag{36}$$

Substituting Eq. (36) into (33), the effectiveness becomes

$$\boxed{\frac{b_e}{b} = \frac{\lambda}{-\lambda - 4 \sum_q \varphi_{0q} q^2 (-1)^q - 4 \sum_q \varphi_{1q} q^2 (-1)^q \cos(2\pi x/a)} \ .} \tag{37}$$

Equation (37) shows that the effective width is not constant over the length of the plate. But we know that

$$-a/2 \leqq x \leqq a/2 \ .$$

Because only symmetrical cases are considered,

$$0 \leqq x \leqq a/2 \ .$$

Therefore,

$$0 \leqq \cos(2\pi x/a) \leqq 1 \ .$$

We can see that the effectiveness of the plate becomes maximum at $x = \pm a/2$ and minimum at $x = 0$. Considering the small amount of change, at $x = \pm a/4$, which is half-way between the center line and edge of the plate, taking only the average value into account, the effectiveness of the plate becomes

$$\boxed{\frac{b_e}{b} = \frac{\lambda}{-\lambda - 4 \sum_q \varphi_{0q} q^2 (-1)^q} \ .} \tag{38}$$

The lower bound of effectiveness can be found by

$$\boxed{\frac{b_e}{b} = \frac{\lambda}{-\lambda - 4 \sum_q \varphi_{0q} q^2 (-1)^q + 4 \sum_q \varphi_{1q} q^2 (-1)^q} ,} \tag{39}$$

and the upper bound of effectiveness can be found by

$$\boxed{\frac{b_e}{b} = \frac{\lambda}{-\lambda - 4 \sum_q \varphi_{0q} q^2 (-1)^q - 4 \sum_q \varphi_{1q} q^2 (-1)^q} .} \tag{40}$$

The results are tabulated in Tables 13 through 17. They are also shown in Figs. 4 through 7.

IV. CONCLUSIONS

1. The effects of initial deflection upon the effective width of a simply supported wide plate in compression are most marked near the theoretical flat-plate critical stress (Fig. 7). At stresses well above or below the critical stress, the behavior of a plate with an initial deflection is very much the same as that of a plate that is initially perfectly flat, provided the center initial deflections are much less than 0.08t.

2. For the same load condition, a wider plate has less effectiveness.

3. Relatively speaking, the degree of initial deflection causes much more reduction of the effectiveness of the plate than the aspect ratio does.

4. The effective width is, at all values of stress, less than that of an initially flat plate.

5. The net center-deflection curves, with various degress of initial deflection (Fig. 3-a), have the same form of the effective width curves (Fig. 7), provided Fig. 7 is rotated 90 degrees counterclockwise.

6. Within the critical-load region, the experimental data closely agree with these theoretical results.

7. For an initially deflected wide plate, its magnitude exceeding 0.1t, the effectiveness is very low, either because of the different assumptions or for some other reason.

8. The effective-width curves have inflection points at critical

loads.

9. The equation of effective width shows that at zero in-plane loads, the effectiveness of the plate must be 100%. In Figs. 4 through 7, the curves seem to start from points other than 100% points, but they are not true. The author surmises that if there is an initial deflection in the plate, the plate has 100% effectiveness if no load is applied, but as soon as any in-plane load is applied to the plate, an interaction starts between initial deflection and additional deflection, causing effectiveness to drop suddenly, depending upon the degree of initial deflection.

10. The effectiveness under the conventional conception is almost independent of the x function (longitudinal). The computed results show that the difference is far less than 1% between maximum and minimum values.

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TABLE 1. φ_{pq} Index Combinations

r = m', s = n' m = m' = 1 always P = 0

q n,n' φ_{pq}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\varphi_{\frac{m-r}{2},\frac{n-s}{2}}$ $\varphi_{\frac{m'-m}{2},\frac{n-n'}{2}}$	3,1 5,3 7,5 9,7 11,9 13,11 15,13	5,1 7,3 9,5 11,7 13,9 15,11	7,1 9,3 11,5 13,7 15,9	9,1 11,3 13,5 15,7	11,1 13,3 15,5	13,1 15,3	15,1								
$\varphi_{\frac{m-m'}{2},\frac{n'-n}{2}}$ $\varphi_{\frac{m'-m}{2},\frac{n'-n}{2}}$	1,3 3,5 5,7 7,9 9,11 11,13 13,15	1,5 3,7 5,9 7,11 9,13 11,15	1,7 3,9 5,11 7,13 9,15	1,9 3,11 5,13 7,15	1,11 3,13 5,15	1,13 3,15	1,15								
$\varphi_{\frac{m-m'}{2},\frac{n+n'}{2}}$ or $\varphi_{\frac{m-r}{2},\frac{n+s}{2}}$ or $\varphi_{\frac{m'-m}{2},\frac{n+n'}{2}}$	1,1	1,3 3,1	1,5 3,3 5,1	1,7 3,5 5,3 7,1	1,9 3,7 5,5 7,3 9,1	1,11 3,9 5,7 7,5 9,3 11,1	1,13 3,11 5,9 7,7 9,5 11,3 13,1	1,15 3,13 5,11 7,9 9,7 11,5 13,3 15,1	3,15 5,13 7,11 9,9 11,7 13,5 15,3	5,15 7,13 9,11 11,9 13,7 15,5	7,15 9,13 11,11 13,9 15,7	9,15 11,13 13,11 15,9	11,15 13,13 15,11	13,15 15,13	15,15

TABLE 2. φ_{pq} Index Combinations

$m = m' = 1$ $P = 1$

q n,n' φ_{pq}	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\varphi_{\frac{m+m'}{2},\frac{n-n'}{2}}$	1,1 3,3 5,5 7,7 9,9 11,11 13,13 15,15	3,1 5,3 7,5 9,7 11,9 13,11 15,13	5,1 7,3 9,5 11,7 13,9 15,11	7,1 9,3 11,5 13,7 15,9	9,1 11,3 13,5 15,7	11,1 13,3 15,5	13,1 15,3	15,1							
$\varphi_{\frac{m'+m}{2},\frac{n'-m}{2}}$	1,1 3,3 5,5 7,7 9,9 11,11 13,13 15,15	1,3 3,5 5,7 7,9 9,11 11,13 13,15	1,5 3,7 5,9 7,11 9,13 11,15	1,7 3,9 5,11 7,13 9,15	1,9 3,11 5,13 7,15	1,11 3,13 5,15	1,13 3,15	1,15							
$\varphi_{\frac{m+m'}{2},\frac{n+n'}{2}}$	0	0	1,3 3,1	1,5 0 5,1	1,7 3,5 5,3 7,1	1,9 3,7 0 7,3 9,1	1,11 3,9 5,7 7,5 9,3 11,1	1,13 3,11 5,9 0 9,5 11,3 13,1	1,15 3,13 5,11 7,9 9,7 11,5 13,3 15,1	3,15 5,13 7,11 0 11,7 13,5 15,3	5,15 7,13 9,11 11,9 13,7 15,5	7,15 9,13 0 13,9 15,7	9,15 11,13 13,11 15,9	11,15 0 15,11	13,15 15,13

TABLE 3. Stress Function Coefficients, φ_{pq}

$$\begin{aligned}\varphi_{01} = {} & 0.5d_{01}[b_1^2 + 2b_1b_3 + 2b_3b_5 + 2b_5b_7 + 2b_7b_9 + 2b_9b_{11} + 2b_{11}b_{13} \\ & + 2b_{13}b_{15} + 2b_1(a_1 + a_3) + 2b_3(a_1 + a_5) + 2b_5(a_3 + a_7) + 2b_7(a_5 + a_9) \\ & + 2b_9(a_7 + a_{11}) + 2b_{11}(a_9 + a_{13}) + 2b_{13}(a_{11} + a_{15}) + 2b_{15}a_{13}] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{02} = {} & 4d_{02}[b_1(b_3 + b_5) + b_7(b_3 + b_{11}) + b_9(b_5 + b_{13}) + b_{11}b_{15} + b_1(a_3 + a_5) \\ & + b_3(a_1 + a_7) + b_5(a_1 + a_9) + b_7(a_3 + a_{11}) + b_9(a_5 + a_{13}) + b_{11}(a_7 + a_{15}) \\ & + b_{13}a_9 + b_{15}a_{11}] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{03} = {} & 4.5d_{03}[b_3(b_3 + 2b_9) + 2b_5(b_1 + b_{11}) + 2b_7(b_1 + b_{13}) + 2b_9b_{15} \\ & + 2b_1(a_5 + a_7) + 2b_3(a_3 + a_9) + 2b_5(a_1 + a_{11}) + 2b_7(a_1 + a_{13}) \\ & + 2b_9(a_3 + a_{15}) + 2b_{11}a_5 + 2b_{13}a_7 + 2b_{15}a_9] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{04} = {} & 16d_{04}[b_1(b_7 + b_9) + b_3(b_5 + b_{11}) + b_5b_{13} + b_7b_{15} + b_1(a_7 + a_9) \\ & + b_3(a_5 + a_{11}) + b_5(a_3 + a_{13}) + b_7(a_1 + a_{15}) + b_9a_1 + b_{11}a_3 + b_{13}a_5 \\ & + b_{15}a_7] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{05} = {} & 12.5d_{05}[2b_1(b_9 + b_{11}) + 2b_3(b_7 + b_{13}) + b_5(b_5 + 2b_{15}) + 2b_1(a_9 + a_{11}) \\ & + 2b_3(a_7 + a_{13}) + 2b_5(a_5 + a_{15}) + 2b_7a_3 + 2b_7a_1 + 2b_{11}a_1 + 2b_{13}a_3 \\ & + 2b_{15}a_5] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{06} = {} & 36d_{06}[b_1(b_{11} + b_{13}) + b_3(b_9 + b_{15}) + b_5b_7 + b_1(a_{11} + a_{13}) + b_3(a_9 + a_{15}) \\ & + b_5a_7 + b_7a_5 + b_9a_3 + b_{11}a_1 + b_{13}a_1 + b_{15}a_3] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{07} = {} & 24.5d_{07}[b_7^2 + 2b_1(b_{13} + b_{15}) + 2b_3b_{11} + 2b_5b_9 + 2b_1(a_{13} + a_{15}) + 2b_3a_{11} \\ & + 2b_5a_9 + 2b_7a_7 + 2b_9a_5 + 2b_{11}a_3 + 2b_{13}a_1 + 2b_{15}a_1] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{08} = {} & 64d_{08}[b_1b_{15} + b_3b_{13} + b_5b_{11} + b_7b_9 + b_1a_{15} + b_3a_{13} + b_5a_{11} + b_7a_9 \\ & + b_9a_7 + b_{11}a_5 + b_{13}a_3 + b_{15}a_1] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{09} = {} & 40.5d_{09}[b_9^2 + 2b_3b_{15} + 2b_5b_{13} + 2b_7b_{11} + 2b_3a_{15} + 2b_5a_{13} + 2b_7a_{11} \\ & + 2b_9a_9 + 2b_{11}a_7 + 2b_{13}a_5 + 2b_{15}a_3] \ .\end{aligned}$$

$$\begin{aligned}\varphi_{0,10} = {} & 100d_{0,10}[b_5b_{15} + b_7b_{13} + b_9b_{11} + b_5a_{15} + b_7a_{13} + b_9a_{11} + b_{11}a_9 \\ & + b_{13}a_7 + b_{15}a_5] \ .\end{aligned}$$

TABLE 3 (continued)

$$\varphi_{0,11} = 60.5d_{0,11}[b_{11}^2 + 2b_7b_{15} + 2b_9b_{13} + 2b_7a_{15} + 2b_9a_{13} + 2b_{11}a_{11} + 2b_{13}a_9 + 2b_{15}a_7] .$$

$$\varphi_{0,12} = 144d_{0,12}[b_9b_{15} + b_{11}b_{13} + b_9a_{15} + b_{11}a_{13} + b_{13}a_{11} + b_{15}a_9] .$$

$$\varphi_{0,13} = 84.5d_{0,13}[b_{13}^2 + 2b_{11}b_{15} + 2b_{11}a_{15} + 2b_{13}a_{13} + 2b_{15}a_{11}] .$$

$$\varphi_{0,14} = 196d_{0,14}[b_{13}b_{15} + b_{13}a_{15} + b_{15}a_{13}] .$$

$$\varphi_{0,15} = 112.5d_{0,15}[b_{15}^2 + 2b_{15}a_{15}] .$$

$$\varphi_{1,0} = 0.5d_{1,0}[b_1^2 + 9b_3^2 + 25b_5^2 + 49b_7^2 + 81b_9^2 + 121b_{11}^2 + 169b_{13}^2 + 225b_{15}^2 + 2b_1a_1 + 18b_3a_3 + 50b_5a_5 + 98b_7a_7 + 162b_9a_9 + 242b_{11}a_{11} + 338b_{13}a_{13} + 450\, b_{15}a_{15}] .$$

$$\varphi_{1,1} = 4d_{1,1}[b_3(b_1 + 4b_5) + b_7(9b_5 + 16b_9) + b_{11}(25b_9 + 36b_{13}) + 49b_{13}b_{15} + b_1a_3 + b_3(a_1 + 4a_5) + b_5(4a_3 + 9a_7 + b_7(9a_5 + 16a_9) + b_9(16a_7 + 25a_{11}) + b_{11}(25a_9 + 36a_{13}) + b_{13}(36a_{11} + 49a_{15}) + 49b_{15}a_{13}] .$$

$$\varphi_{1,2} = d_{1,2}[b_1(b_3 + 9b_5) + b_7(25b_3 + 81b_{11}) + b_9(49b_5 + 121b_{13}) + 169b_{11}b_{15} + b_1(a_3 + 9a_5) + b_3(a_1 + 25a_7) + b_5(9a_1 + 49a_9) + b_7(25a_3 + 81a_{11}) + b_9(49a_5 + 121a_{13}) + b_{11}(81a_7 + 169a_{15}) + 121a_9b_{13} + 169b_{15}a_{11}] .$$

$$\varphi_{1,3} = 4d_{1,3}[b_5(b_1 + 16b_{11}) + b_7(4b_1 + 25b_{13}) + b_9(9b_3 + 36b_{15}) + b_1(a_5 + 4a_7) + 9b_3a_9 + b_5(a_1 + 16a_{11}) + b_7(4a_1 + 25a_{13}) + b_9(9a_3 + 36a_{15}) + 16a_5b_{11} + 25a_7b_{13} + 36a_9b_{15}] .$$

$$\varphi_{1,4} = d_{1,4}[b_1(9b_7 + 25b_9) + b_3(b_5 + 49b_{11}) + 81b_5b_{13} + 121b_7b_{15} + b_1(9a_7 + 25a_9) + b_3(a_5 + 49a_{11}) + b_5(a_3 + 81a_{13}) + b_7(9a_1 + 121a_{15}) + 25a_1b_9 + 49a_3b_{11} + 81a_5b_{13} + 121a_7b_{15}] .$$

$$\varphi_{1,5} = 4d_{1.5}[b_1(4b_9 + 9b_{11}) + b_3(b_7 + 16b_{13}) + 25\, b_5b_{15} + b_1(4a_9 + 9a_{11}) + b_3(a_7 + 16a_{13}) + 25b_5a_{15} + b_7a_3 + 4b_9a_1 + 9b_{11}a_1 + 16b_{13}a_3 + 25b_{15}a_5].$$

TABLE 3 (continued)

$$\varphi_{1,6} = d_{1,6}[b_1(25b_{11} + 49b_{13}) + b_3(9b_9 + 81b_{15}) + b_5b_7 + b_1(49a_{13} + 25a_{11}) + b_3(81a_{15} + 9a_9) + b_5a_7 + b_7a_5 + 9b_9a_3 + 25b_{11}a_1 + 49b_{13}a_1 + 81b_{15}a_3] .$$

$$\varphi_{1,7} = 4d_{1,7}[b_1(9b_{13} + 16b_{15}) + 4b_3b_{11} + b_5b_9 + b_1(16a_{15} + 9a_{13}) + 4b_3a_{11} + b_5a_9 + b_9a_5 + 4b_{11}a_3 + 9b_{13}a_1 + 16b_{15}a_1] .$$

$$\varphi_{1,8} = d_{1,8}[49b_1b_{15} + 25b_3b_{13} + 9b_5b_{11} + b_7b_9 + 49b_1a_{15} + 25b_3a_{13} + 9b_5a_{11} + b_7a_9 + b_9a_7 + 9b_{11}a_5 + 25b_{13}a_3 + 49b_{15}a_1] .$$

$$\varphi_{1,9} = 4d_{1,9}(9b_3b_{15} + 4b_5b_{13} + b_7b_{11} + 9b_3a_{15} + 4b_5a_{13} + b_7a_{11} + b_{11}a_7 + 4b_{13}a_5 + 9b_{15}a_3) .$$

$$\varphi_{1,10} = d_{1,10}(25b_5b_{15} + 9b_7b_{13} + b_9b_{11} + 25b_5a_{15} + 9b_7a_{13} + b_9a_{11} + b_{11}a_9 + 9b_{13}a_7 + 25b_{15}a_5) .$$

$$\varphi_{1,11} = 4d_{1,11}(4b_7b_{15} + b_9b_{13} + 4b_7a_{15} + b_9a_{13} + b_{13}a_9 + 4b_{15}a_7) .$$

$$\varphi_{1,12} = d_{1,12}(9b_9b_{15} + b_{11}b_{13} + 9b_9a_{15} + b_{11}a_{13} + b_{13}a_{11} + 9b_{15}a_9) .$$

$$\varphi_{1,13} = 4d_{1,13}(b_{11}b_{15} + b_{11}a_{15} + b_{15}a_{11}) .$$

$$\varphi_{1,14} = d_{1,14}(b_{13}b_{15} + b_{13}a_{15} + b_{15}a_{13}) .$$

$$\varphi_{1,15} = 0 .$$

$$d_{pq} = - \frac{\beta^2}{16(\beta^2p^2 + q^2)^2} .$$

NOTE: In b_{mn}, $m = 1$ for all cases. For clarity $m = 1$ has been left out.

Example: $b_{1,n}$ is represented by b_n.

TABLE 4. Values of Coefficients in Initial Deflection Function

$\beta = 3.5 \qquad \nu = 0.3$

pa^4/Et^4	$a_{1,1}/t$	$a_{1,3}/t$	$a_{1,5}/t$	$a_{1,7}/t$	$a_{1,9}/t$	$a_{1,11}/t$	$a_{1,13}/t$	$a_{1,15}/t$	$w_{o;center}/t$
0.0	0.00000	-0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0
0.64620	0.10000	-0.01333	0.00260	-0.00068	0.00022	-0.00009	0.00004	-0.00002	0.08874
1.3064	0.20000	-0.02875	0.00567	-0.00144	0.00047	-0.00018	0.00008	-0.00004	0.17581
1.9919	0.30000	-0.04777	0.00976	-0.00240	0.00075	-0.00029	0.00013	-0.00006	0.26012
2.7096	0.40000	-0.07101	0.01546	-0.00374	0.00111	-0.00041	0.00018	-0.00009	0.34150
3.4634	0.50000	-0.09836	0.02328	-0.00567	0.00161	-0.00056	0.00024	-0.00011	0.42043
7.8050	1.00000	-0.27192	0.09715	-0.03128	0.00945	-0.00291	0.00098	-0.00047	0.80100

TABLE 5. Values of Initial Deflection Coefficients

$\beta = 5.0 \qquad \nu = 0.3$

pa^4/Et^4	$a_{1,1}/t$	$a_{1,3}/t$	$a_{1,5}/t$	$a_{1,7}/t$	$a_{1,9}/t$	$a_{1,11}/t$	$a_{1,13}/t$	$a_{1,15}/t$	$w_{o,center}/t$
0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0
0.59705	0.10000	-0.01993	0.00560	-0.00182	0.00068	-0.00029	0.00013	-0.00007	0.08430
1.2048	0.20000	-0.04226	0.01237	-0.00401	0.00147	-0.00061	0.00029	-0.00015	0.16710
1.8308	0.30000	-0.06834	0.02140	-0.00705	0.00253	-0.00102	0.00046	-0.00023	0.24775
2.4789	0.40000	-0.09829	0.03349	-0.01150	0.00410	-0.00159	0.00069	-0.00035	0.32655
3.1501	0.50000	-0.13134	0.04884	-0.01785	0.00647	-0.00246	0.00103	-0.00051	0.40418
6.8299	1.00000	-0.31317	0.15660	-0.07850	0.03665	-0.01613	0.00691	-0.00474	0.78762

TABLE 6. Values of Initial Deflection Coefficients

$B = 6.5 \qquad \nu = 0.3$

pa^4/Et^4	$a_{1,1}/t$	$a_{1,3}/t$	$a_{1,5}/t$	$a_{1,7}/t$	$a_{1,9}/t$	$a_{1,11}/t$	$a_{1,13}/t$	$a_{1,15}/t$	$w_{o,center}/t$
0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0	-0.0	0.0
0.57814	0.10000	-0.02415	0.00856	-0.00333	0.00141	-0.00065	0.00032	-0.00017	0.08199
1.1648	0.20000	-0.05052	0.01881	-0.00749	0.00316	-0.00143	0.00070	-0.00037	0.16286
1.7656	0.30000	-0.08013	0.03200	-0.01333	0.00567	-0.00253	0.00121	-0.00063	0.24226
2.3824	0.40000	-0.11271	0.04859	-0.02158	0.00949	-0.00423	0.00196	-0.00105	0.32047
3.0148	0.50000	-0.14731	0.06817	-0.03250	0.01505	-0.00686	0.00316	-0.00178	0.39793
6.3790	1.00000	-0.32645	0.18322	-0.11196	0.06683	-0.03793	0.02056	-0.01906	0.77521

TABLE 7. Additional Deflection Coefficients b_{mn} Values

$w_{o,center}/t = 0.0$ $\beta = 3.5$, $\nu = 0.3$

$\sigma_o b^2/\pi^2 Et^2$ (ψ)	$b_{1,1}/t$	$b_{1,3}/t$	$b_{1,5}/t$	$b_{1,7}/t$	$b_{1,9}/t$	$b_{1,11}/t$	$b_{1,13}/t$	$b_{1,15}/t$	w_{center}/t
2.647900 (2.0179)	2.000000	-.496340	.152110	-.040052	.009545	-.002186	.000491	-.000108	1.623460
2.440100 (1.8595	1.800000	-.430820	.122810	-.029690	.006510	-.001373	.000284	-.000057	1.467644
2.249400 (1.7142)	1.600000	-.364640	.094718	-.020620	.004085	-.000779	.000145	-.000026	1.312883
2.074900 (1.5812)	1.400000	-.298070	.068584	-.013102	.002287	-.000384	.000063	-.000026	1.159352
1.915900 (1.4600)	1.200000	-.231790	.045404	-.007343	.001090	-.000155	.000021	-.000010	1.007217
1.771300 (1.3498)	1.000000	-.167220	.026333	-.003421	.000409	-.000047	.000005	-.000002	.856057
1.640100 (1.2498)	.800000	-.107090	.012436	-.001197	.000106	-.000009	.0	-.0	.704246
1.521700 (1.1596)	.600000	-.056004	.004170	-.000259	.000014	-.0	.0	-.0	.547921
1.418600 (1.0810)	.400000	-.019965	.000738	-.000023	.0	-.0	.0	-.0	.380750
1.34200 (1.0227)	.200000	-.002835	.000028	-.0	.0	-.0	.0	-.0	.197193
1.320000 (1.0059)	.100000	-.000366	.0	-.0	.0	-.0	.0	-.0	.099634
1.314300 (1.0016)	.050000	-.000046	.0	-.0	.0	-.0	.0	-.0	.049954
1.312700 (1.0003)	.020000	-.000002	.0	-.0	.0	-.0	.0	-.0	.019998

TABLE 8. Additional Deflection Coefficients b_{mn} Values

$w_{o,center}/t = 0.08874$ $\beta = 3.5,\ \nu = 0.3$

$\sigma_o b^2/\pi^2 Et^2$ (ψ)	$b_{1,1}/t$	$b_{1,3}/t$	$b_{1,5}/t$	$b_{1,7}/t$	$b_{1,9}/t$	$b_{1,11}/t$	$b_{1,13}/t$	$b_{1,15}/t$	w_{center}/t
2.684800 (2.0460)	2.000000	-.517680	.165790	-.045586	.011320	-.002698	.000631	-.000145	1.611632
2.461500 (1.8758)	1.800000	-.452810	.136250	-.034721	.008004	-.001773	.000385	-.000082	1.455253
2.253800 (1.7175)	1.600000	-.387320	.107650	-.025015	.005275	-.001070	.000212	-.000041	1.299691
2.060600 (1.5703)	1.400000	-.321390	.080623	-.016730	.003164	-.000576	.000102	-.000018	1.145175
1.880000 (1.4327)	1.200000	-.255500	.056054	-.010104	.001670	-.000266	.000041	-.000006	.991889
1.709300 (1,3026)	1.000000	-.190710	.035021	-.005282	.000736	-.000099	.000013	-.000001	.839678
1.544400 (1,1769)	.800000	-.129080	.018654	-.002237	.000252	-.000028	.000003	-.0	.687564
1.377400 (1,0496)	.600000	-.074290	.007746	-.000694	.000061	-.000005	.0	-.0	.532818
1.191100 (0.9077)	.400000	-.031831	.002144	-.000140	.000010	-.000001	.0	-.0	.370182
0.925950 (0.7056)	.200000	-.007141	.000303	-.000018	.000001	-.0	.0	-.0	.193145
0.675340 (0.5146)	.100000	-.001799	.000073	-.000005	.0	-.0	.0	-.0	.098269
0.445260 (0.3392)	.050000	-.000551	.000025	-.000001	.0	-.0	.0	-.0	.049473
0.221400 (0.1687)	.020000	-.000151	.000008	-.0	.0	-.0	.0	-.0	.019857

TABLE 9. Additional Deflection Coefficients b_{mn} Values

$w_{o,center}/t = 0.0$ $\beta = 5.0,\ \nu = 0.3$

$\sigma_o b^2/\pi^2 Et^2$ (ψ)	$b_{1,1}/t$	$b_{1,3}/t$	$b_{1,5}/t$	$b_{1,7}/t$	$b_{1,9}/t$	$b_{1,11}/t$	$b_{1,13}/t$	$b_{1,15}/t$	w_{center}/t
4.417600 (1.7840)	2.000000	-.580290	.254670	-.109640	.043513	-.016203	.005825	-.002060	1.595815
4.124000 (1,6654)	1.800000	-.512930	.216620	-.088251	.032924	-.011531	.003904	-.001300	1.439436
3.854200 (1.5564)	1.600000	-.444720	.178370	-.067687	.023388	-.007598	.002390	-.000739	1.283404
3.606900 (1.4566)	1.400000	-.375540	.140310	-.048527	.015220	-.004500	.001290	-.000363	1.127890
3.380900 (1.3653)	1.200000	-.305350	.103210	-.031548	.008731	-.002285	.000581	-.000145	0.973194
3.174700 (1,2820)	1.000000	-.234370	.068460	-.017677	.004139	-.000919	.000198	-.000042	0.819789
2.986700 (1.2061)	.800000	-.163600	.038321	-.007789	.001442	-.000254	.000043	-.000007	0.668156
2.815600 (1.1370)	.600000	-.096127	.015851	-.002246	.000291	-.000036	.000004	-.0	0.517737
2.661200 (1.0747)	.400000	-.039512	.003563	-.000276	.000019	-.000001	.0	-.0	0.363793
2.533000 (1.0229)	.200000	-.006412	.000169	-.000003	.0	-.0	.0	-.0	0.193754
2.491400 (1.0061)	.100000	-.000865	.000005	-.0	.0	-.0	.0	-.0	0.099140
2.480100 (1.0015)	.050000	-.000110	.0	-.0	.0	-.0	.0	-.0	0.049890
2.476800 (1,0002)	.020000	-.000007	.0	-.0	.0	-.0	.0	-.0	0.019993

TABLE 10. Additional Deflection Coefficients b_{mn} Values

$w_{o,center}/t = 0.08430$ $\beta = 5.0,\ \nu = 0.3$

$\sigma_o b^2/\pi^2 E t^2$ (ψ)	$b_{1,1}/t$	$b_{1,3}/t$	$b_{1,5}/t$	$b_{1,7}/t$	$b_{1,9}/t$	$b_{1,11}/t$	$b_{1,13}/t$	$b_{1,15}/t$	w_{center}/t
4.438000 (1.7922)	2.000000	-.595120	.269270	-.119610	.048981	-.018797	.006959	-.002535	1.589148
4.119200 (1.6635)	1.800000	-.528360	.231610	-.098091	.038031	-.013819	.004850	-.001674	1.432547
3.821500 (1.5432)	1.600000	-.460920	.193710	-.077204	.027991	-.009521	.003134	-.001013	1.276177
3.542700 (1.4307)	1.400000	-.392700	.155850	-.057434	.019160	-.006011	.001828	-.000546	1.120147
3.279600 (1.3244)	1.200000	-.323670	.118620	-.039458	.011853	-.003360	.000926	-.000250	0.964661
3.027500 (1.2226)	1.000000	-.254000	.083085	-.024124	.006335	-.001578	.000383	-.000091	0.810010
2.778800 (1.1222)	0.800000	-.184380	.051060	-.012341	.002719	-.000574	.000119	-.000024	0.656579
2.518800 (1.0172)	0.600000	-.116880	.025195	-.004744	.000833	-.000144	.000025	-.000004	0.504281
2.212800 (0.8936)	0.400000	-.056752	.008361	-.001140	.000157	-.000023	.000004	-.0	0.350607
1.740300 (0.7028)	0.200000	-.014512	.001287	-.000139	.000019	-.000003	.0	-.0	0.186652
1.272500 (0.5138)	0.100000	-.003823	.000294	-.000035	.000005	-.000001	.0	-.0	0.096440
0.839340 (0.3389)	0.050000	-.001182	.000098	-.000012	.000002	-.0	.0	-.0	0.048906
0.417420 (0.1685)	0.020000	-.000325	.000030	-.000004	.0	-.0	.0	-.0	0.019701

TABLE 11. Additional Deflection Coefficients b_{mn} Values

$w_{o,center}/t = 0.0$ $\beta = 6.5,\ \nu = 0.3$

$\sigma_o b^2/\pi^2 E t^2$ (ψ)	$b_{1,1}/t$	$b_{1,3}/t$	$b_{1,5}/t$	$b_{1,7}/t$	$b_{1,9}/t$	$b_{1,11}/t$	$b_{1,13}/t$	$b_{1,15}/t$	w_{center}/t
6.615000 (1.6317)	2.000000	-.615030	.309260	-.165350	.085707	-.042227	.019927	-.009147	1.583140
6.232400 (1.5373)	1.800000	-.547690	.269270	-.139070	.069006	-.032413	.014573	-.006375	1.427301
5.881200 (1.4507)	1.600000	-.479690	.228650	-.112760	.052896	-.023409	.009919	-.004091	1.271515
5.559700 (1.3714)	1.400000	-.410840	.187450	-.086802	.037833	-.015522	.006105	-.002339	1.115885
5.265900 (1.2989)	1.200000	-.340940	.145900	-.061874	.024446	-.009087	.003246	-.001130	0.960561
4.997400 (1.2327)	1.000000	-.269790	.104690	-.039125	.013508	-.004395	.001377	-.000421	0.805844
4.751900 (1.1721)	0.800000	-.197440	.065472	-.020259	.005765	-.001551	.000403	-.000102	0.652288
4.527400 (1.1168)	0.600000	-.125020	.031659	-.007270	.001536	-.000308	.000059	-.000011	0.500645
4.322800 (1.0663)	0.400000	-.057637	.008798	-.001198	.000150	-.000018	.000002	-.0	0.350097
4.144100 (1.0222)	0.200000	-.010783	.000532	-.000023	.0	-.0	.0	-.0	0.139726
4.079600 (1.0063)	0.100000	-.001536	.000020	-.0	.0	-.0	.0	-.0	0.098484
4.060900 (1.0017)	0.050000	-.000199	.0	-.0	.0	-.0	.0	-.0	0.049801
4.055400 (1.0003)	0.020000	-.000012	.0	-.0	.0	-.0	.0	-.0	0.019988

TABLE 12. Additional Deflection Coefficients b_{mn} Values

$w_{o,center}/t = 0.08199$, $a_{1,1}/t = 0.10000$ $\beta = 6.5$, $\nu = 0.3$

$\sigma_o b^2/\pi^2 Et^2$ (ψ)	$b_{1,1}/t$	$b_{1,3}/t$	$b_{1,5}/t$	$b_{1,7}/t$	$b_{1,9}/t$	$b_{1,11}/t$	$b_{1,13}/t$	$b_{1,15}/t$	w_{center}/t
6.601500 (1.6284)	2.000000	-.625180	.321490	-.175980	.093436	-.047166	.022798	-.010718	1.578680
6.181300 (1.5247)	1.800000	-.558290	.281990	-.149950	.076665	-.037102	.017177	-.007733	1.422667
5.788200 (1.4278)	1.600000	-.490880	.241960	-.123830	.060340	-.027723	.012182	-.005208	1.266841
5.418600 (1.3366)	1.400000	-.422820	.201420	-.097903	.044839	-.019306	.007957	-.003193	1.110994
5.067500 (1.2500)	1.200000	-.353990	.160530	-.072701	.030701	-.012174	.004631	-.001717	0.955280
4.727400 (1.1661)	1.000000	-.284300	.119790	-.049130	.018625	-.006639	.002278	-.000763	0.799861
4.385700 (1.0818)	0.800000	-.213830	.080370	-.028600	.009370	-.002902	.000870	-.000255	0.645023
4.018600 (0.9912)	0.600000	-.143390	.044758	-.012949	.003477	-.000896	.000227	-.000056	0.491171
3.569000 (0.8803)	0.400000	-.076141	.017366	-.003733	.000780	-.000165	.000036	-.000008	0.338135
2.834700 (0.6992)	0.200000	-.022134	.003055	-.000485	.000090	-.000019	.000004	-.000001	0.180510
2.079300 (0.5129)	0.100000	-.006216	.000689	-.000112	.000022	-.000005	.000001	-.0	0.094379
1.372800 (0.3386)	0.050000	-.001968	.000222	-.000039	.000008	-.000002	.0	-.0	0.048221
0.682930 (0.1684)	0.020000	-.000545	.000068	-.000012	.000002	-.0	.0	-.0	0.019513

TABLE 13. Effective Width

$\psi = \lambda/\lambda_{cr} = \lambda/1.3122$ for $\beta = 3.5$

β	$w_{o,center}/t$	λ	ψ	Effectiveness, ρ		
				Max.	Average	Min.
3.5	0.0	2.6479	2.0179	0.29404	0.28819	0.28256
		2.4401	1.8595	0.32168	0.31648	0.31145
		2.2494	1.7142	0.35662	0.35216	0.34781
		2.0749	1.5812	0.40116	0.39752	0.39393
		1.9159	1.4600	0.45825	0.45547	0.45273
		1.7713	1.3498	0.53121	0.52931	0.52742
		1.6401	1.2498	0.62284	0.62174	0.62065
		1.5217	1.1596	0.73285	0.73237	0.73189
		1.4186	1.0810	0.85254	0.85242	0.85229
		1.3420	1.0227	0.95635	0.95634	0.95633
		1.3200	1.0059	0.98853	0.98853	0.98853
		1.3143	1.0016	0.99710	0.99710	0.99710
		1.3127	1.0003	0.99953	0.99953	0.99953
	0.08874	2.6848	2.0460	0.27736	0.27117	0.26525
		2.4615	1.8758	0.30081	0.29522	0.28982
		2.2538	1.7175	0.33020	0.32527	0.32049
		2.0606	1.5703	0.36742	0.36323	0.35913
		1.8800	1.4327	0.41492	0.41153	0.40819
		1.7093	1.3026	0.47573	0.47316	0.47062
		1.5444	1.1769	0.55317	0.55140	0.54964
		1.3774	1.0496	0.64957	0.64848	0.64739
		1.1911	0.9077	0.76340	0.76280	0.76221
		0.9259	0.7056	0.88314	0.88285	0.88255
		0.6753	0.5146	0.93640	0.93621	0.93602
		0.4452	0.3392	0.95888	0.95874	0.95859
		0.2214	0.1687	0.97056	0.97044	0.97032
	0.17581	2.7261	2.0775	0.26321	0.25672	0.25055
		2.4882	1.8962	0.28325	0.27731	0.27161
		2.2650	1.7261	0.30818	0.30284	0.29768
		2.0550	1.5660	0.33951	0.33484	0.33030
		1.8558	1.4142	0.37926	0.37532	0.37146
		1.6639	1.2680	0.43002	0.42684	0.42371
		1.4737	1.1230	0.49496	0.49255	0.49016
		1.2750	0.9716	0.57734	0.57562	0.57392
		1.0461	0.7972	0.67909	0.67795	0.67681
		0.7256	0.5529	0.79746	0.79674	0.79603
		0.4667	0.3556	0.85902	0.85847	0.85792
		0.2755	0.2099	0.88892	0.88846	0.88799
		0.1241	0.0945	0.90622	0.90581	0.90539

TABLE 14. Effective Width

β	$w_{o,center}/t$	λ	ψ	Effectiveness, ρ		
				Max.	Average	Min.
3.5 (cont.)	0.26012	2.7711	2.1117	0.25108	0.24432	0.23791
		2.5192	1.9198	0.26832	0.26205	0.25607
		2.2815	1.7386	0.28960	0.28388	0.27839
		2.0561	1.5669	0.31614	0.31104	0.30610
		1.8404	1.4025	0.34959	0.34515	0.34082
		1.6306	1.2426	0.39210	0.38836	0.38469
		1.4206	1.0826	0.44646	0.44343	0.44044
		1.1996	0.9141	0.51595	0.51360	0.51127
		0.9466	0.7213	0.60388	0.60213	0.60039
		0.6098	0.4647	0.71194	0.71068	0.70942
		0.3659	0.2788	0.77275	0.77170	0.77065
		0.2050	0.1562	0.80431	0.80336	0.80241
		0.0886	0.0675	0.82344	0.82256	0.82168
	0.42043	2.8699	2.1870	0.23148	0.22419	0.21734
		2.5918	1.9752	0.24444	0.23756	0.23107
		2.3271	1.7734	0.26020	0.25379	0.24769
		2.0739	1.5804	0.27958	0.27367	0.26800
		1.8295	1.3942	0.30364	0.29826	0.29308
		1.5901	1.2117	0.33382	0.32901	0.32434
		1.3498	1.0286	0.37204	0.36782	0.36369
		1.0992	0.8376	0.42086	0.41721	0.41362
		0.8217	0.6262	0.48357	0.48045	0.47738
		0.4836	0.3685	0.58411	0.56148	0.55887
		0.2696	0.2054	0.61242	0.61000	0.60761
		0.1437	0.1095	0.63879	0.63649	0.63421
		0.0600	0.0457	0.65536	0.65313	0.65092
5.0	0.0	4.4176	1.7840	0.23058	0.22574	0.22110
		4.1240	1.6654	0.25379	0.24935	0.24506
		3.8542	1.5564	0.28313	0.27916	0.27531
		3.6069	1.4566	0.32065	0.31724	0.31391
		3.3809	1.3653	0.36915	0.36638	0.36366
		3.1747	1.2820	0.43228	0.43020	0.42813
		2.9867	1.2061	0.51424	0.51283	0.51142
		2.8156	1.1370	0.61833	0.61750	0.61667
		2.6612	1.0747	0.74287	0.74243	0.74200
		2.5330	1.0229	0.87350	0.87329	0.87307
		2.4914	1.0061	0.93116	0.93102	0.93088
		2.4801	1.0015	0.95546	0.95536	0.95526
		2.4768	1.0002	0.96810	0.96802	0.96793

TABLE 15. Effective Width

$\lambda_{cr} = 2.4762$ for $\beta = 5.0$

β	$w_{o,center}/t$	λ	ψ	Effectiveness, ρ		
				Max.	Average	Min.
5.0 (cont.)	0.08430	4.4380	1.7922	0.23058	0.22574	0.22110
		4.1192	1.6635	0.25379	0.24935	0.24506
		3.8215	1.5432	0.28313	0.27916	0.27531
		3.5427	1.4307	0.32065	0.31724	0.31391
		3.2796	1.3244	0.36915	0.36638	0.36366
		3.0275	1.2226	0.43228	0.43020	0.42813
		2.7788	1.1222	0.51424	0.51283	0.51142
		2.5188	1.0172	0.61833	0.61750	0.61667
		2.2128	0.8936	0.74287	0.74243	0.74200
		1.7403	0.7028	0.87350	0.87329	0.87307
		1.2725	0.5138	0.93116	0.93102	0.93088
		0.8393	0.3389	0.95546	0.95536	0.95526
		0.4174	0.1685	0.96810	0.96802	0.96793
	0.16710	4.4674	1.8041	0.21660	0.21159	0.20681
		4.1252	1.6659	0.23633	0.23167	0.22719
		3.8024	1.5355	0.26104	0.25680	0.25269
		3.4959	1.4118	0.29236	0.28861	0.28495
		3.2017	1.2929	0.33254	0.32935	0.32622
		2.9134	1.1765	0.38460	0.38202	0.37948
		2.6205	1.0582	0.45237	0.45042	0.44849
		2.3039	0.9304	0.54007	0.53871	0.53736
		1.9218	0.7761	0.65045	0.64956	0.64868
		1.3533	0.5465	0.77990	0.77935	0.77880
		0.8751	0.3534	0.84697	0.84655	0.84614
		0.5175	0.2089	0.87945	0.87910	0.87875
		0.2333	0.0942	0.89822	0.89791	0.89760
	0.24775	4.5043	1.8190	0.20467	0.19951	0.19461
		4.1402	1.6719	0.22156	0.21671	0.21207
		3.7942	1.5322	0.24254	0.23806	0.23373
		3.4630	1.3985	0.26890	0.26484	0.26090
		3.1418	1.2687	0.30241	0.29884	0.29536
		2.8236	1.1402	0.34554	0.34251	0.33954
		2.4970	1.0083	0.40152	0.39906	0.39663
		2.1422	0.8651	0.47445	0.47255	0.47066
		1.7188	0.6941	0.56862	0.56721	0.56580
		1.1259	0.4546	0.68622	0.68520	0.68419
		0.6806	0.2748	0.75265	0.75181	0.75097
		0.3826	0.1545	0.78708	0.78633	0.78558
		0.1657	0.0669	0.80793	0.80723	0.80653

TABLE 16. Effective Width

β	$w_{o,center}/t$	λ	ψ	Effectiveness, ρ		
				Max.	Average	Min.
5.0 (cont.)	0.40418	4.5964	1.8562	0.18550	0.18006	0.17492
		4.1916	1.6927	0.19812	0.19291	0.18796
		3.8035	1.5360	0.21352	0.20859	0.20387
		3.4284	1.3845	0.23254	0.22792	0.22348
		3.0616	1.2364	0.25632	0.25205	0.24792
		2.6960	1.0887	0.28640	0.28253	0.27876
		2.3207	0.9372	0.32493	0.32147	0.31809
		1.9182	0.7746	0.37486	0.37183	0.36885
		1.4571	0.5884	0.44017	0.43754	0.43494
		0.8725	0.3523	0.52579	0.52353	0.52128
		0.4909	0.1982	0.57786	0.57577	0.57369
		0.2630	0.1062	0.60645	0.60445	0.60247
		0.1100	0.0444	0.62445	0.62251	0.62058
6.5	0.0	6.6150	1.6317	0.22095	0.21733	0.21382
		6.2324	1.5373	0.24830	0.24494	0.24167
		5.8812	1.4507	0.28336	0.28035	0.27741
		5.5597	1.3714	0.32888	0.32633	0.32383
		5.2659	1.2989	0.38858	0.38660	0.38464
		4.9974	1.2327	0.46708	0.46574	0.46440
		4.7519	1.1721	0.56895	0.56822	0.56749
		4.5274	1.1168	0.69500	0.69473	0.69446
		4.3228	1.0663	0.83364	0.83359	0.83354
		4.1441	1.0222	0.95136	0.95136	0.95136
		4.0796	1.0063	0.98722	0.98722	0.98722
		4.0609	1.0017	0.99676	0.99676	0.99676
		4.0554	1.0003	0.99948	0.99948	0.99948
	0.08199	6.6015	1.6284	0.20453	0.20083	0.19727
		6.1813	1.5247	0.22750	0.22401	0.22063
		5.7882	1.4278	0.25665	0.25345	0.25032
		5.4186	1.3366	0.29414	0.29131	0.28853
		5.0675	1.2500	0.34292	0.34057	0.33824
		4.7274	1.1661	0.40697	0.40517	0.40338
		4.3857	1.0818	0.49102	0.48980	0.48858
		4.0186	0.9912	0.59917	0.59846	0.59776
		3.5690	0.8803	0.73022	0.72987	0.72952
		2.8347	0.6992	0.86817	0.86800	0.86784
		2.0793	0.5129	0.92853	0.92843	0.92832
		1.3728	0.3386	0.95383	0.95376	0.95368
		0.6829	0.1684	0.96695	0.96689	0.96683

TABLE 17. Effective Width

λ_{cr} = 4.0539 for β = 6.5

β	$w_{o,center}/t$	λ	ψ	Effectiveness, ρ		
				Max.	Average	Min.
6.5 (cont.)	0.16286	6.6034	1.6289	0.19072	0.18696	0.18334
		6.1489	1.5167	0.21018	0.20659	0.20312
		5.7183	1.4105	0.23465	0.23129	0.22802
		5.3074	1.3092	0.26580	0.26275	0.25977
		4.9090	1.2109	0.30600	0.30333	0.30072
		4.5131	1.1132	0.35845	0.35626	0.35409
		4.1028	1.0120	0.42739	0.42572	0.42406
		3.6467	0.8995	0.51767	0.51650	0.51534
		3.0759	0.7587	0.63280	0.63206	0.63131
		2.1904	0.5403	0.76931	0.76886	0.76841
		1.4235	0.3511	0.84009	0.83976	0.83942
		0.8435	0.2080	0.87426	0.87398	0.87370
		0.3807	0.0939	0.89395	0.89371	0.89346
	0.24226	6.6184	1.6326	0.17896	0.17516	0.17151
		6.1320	1.5126	0.19559	0.19191	0.18838
		5.6676	1.3980	0.21629	0.21280	0.20942
		5.2197	1.2875	0.24240	0.23915	0.23600
		4.7810	1.1793	0.27576	0.27284	0.26997
		4.3401	1.0705	0.31897	0.31643	0.31393
		3.8785	0.9567	0.37552	0.37342	0.37135
		3.3363	0.8229	0.44994	0.44831	0.44668
		2.7294	0.6732	0.54726	0.54605	0.54484
		1.8090	0.4462	0.67041	0.66954	0.66868
		1.1001	0.2713	0.74047	0.73976	0.73905
		0.6201	0.1529	0.77683	0.77619	0.77556
		0.2691	0.0663	0.79884	0.79825	0.79767
	0.39793	6.6976	1.6476	0.16013	0.15623	0.15251
		6.1347	1.5132	0.17248	0.16865	0.16499
		5.6094	1.3837	0.18760	0.18388	0.18030
		5.0980	1.2575	0.20633	0.20275	0.19929
		4.5928	1.1329	0.22981	0.22642	0.22312
		4.0823	1.0070	0.25967	0.25650	0.25341
		3.5491	0.8754	0.29815	0.29525	0.29241
		2.9646	0.7312	0.34841	0.34580	0.34324
		2.2772	0.5617	0.41479	0.41248	0.41020
		1.3801	0.3404	0.50291	0.50087	0.49886
		0.7816	0.1928	0.55702	0.55513	0.55325
		0.4200	0.1036	0.58688	0.58507	0.58326
		0.1761	0.0434	0.60573	0.60396	0.60221

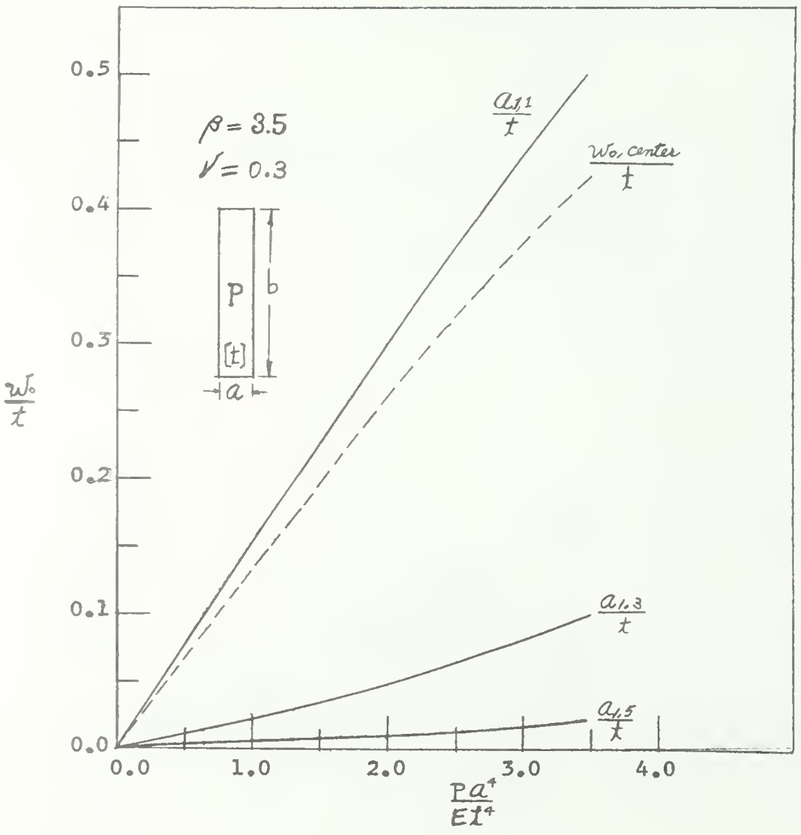

Fig. 1. INITIAL DEFLECTION COEFFICIENTS

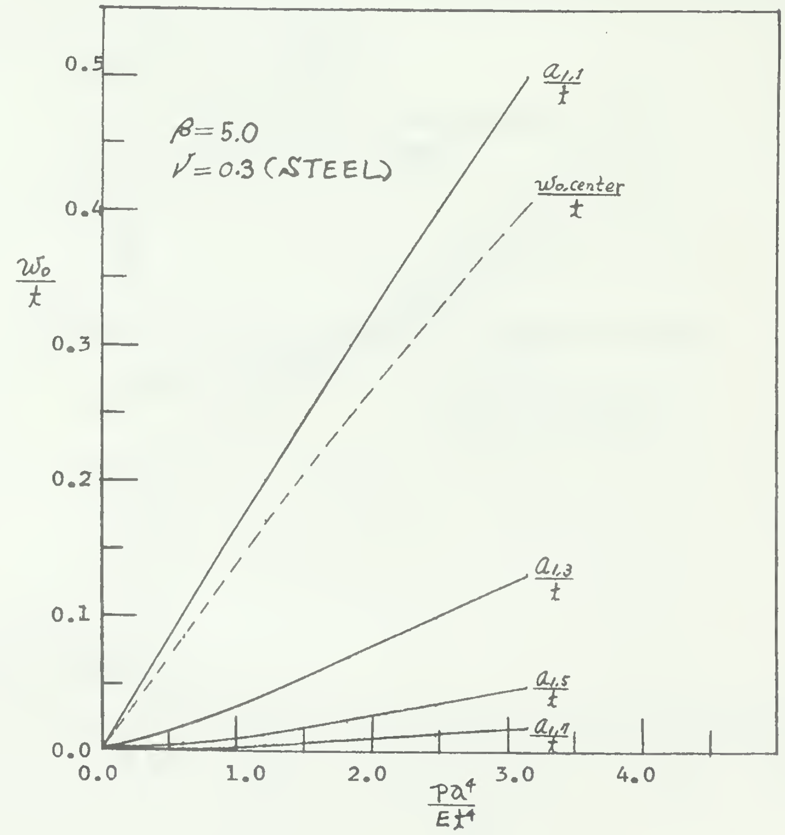

Fig. 2. INITIAL DEFLECTION COEFFICIENTS

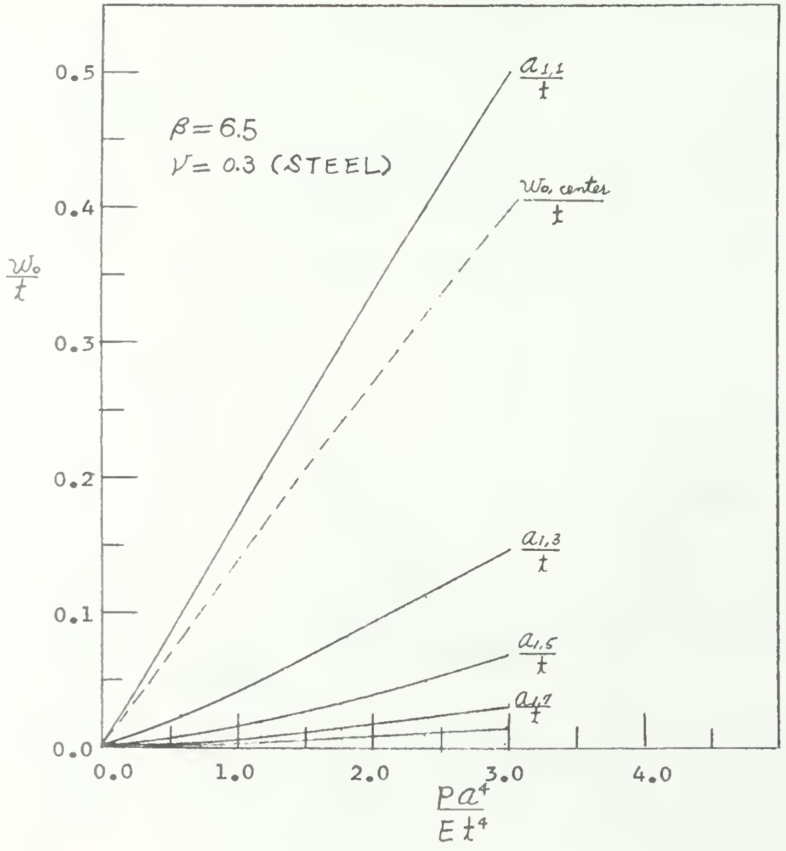

Fig. 3. INITIAL DEFLECTION COEFFICIENTS

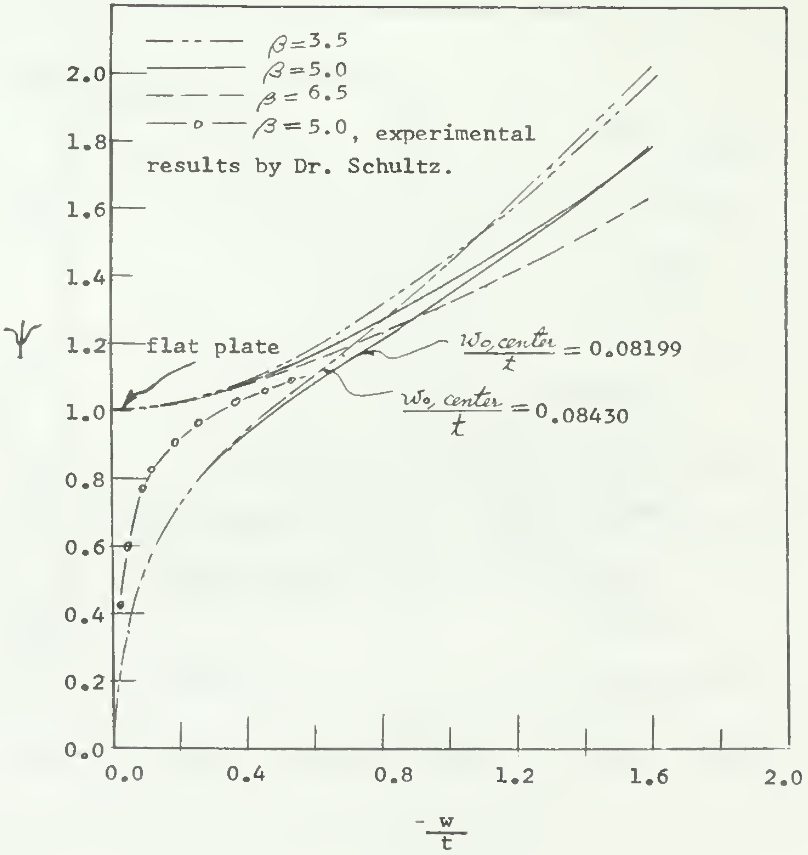

Fig. 3-a. Net center deflections

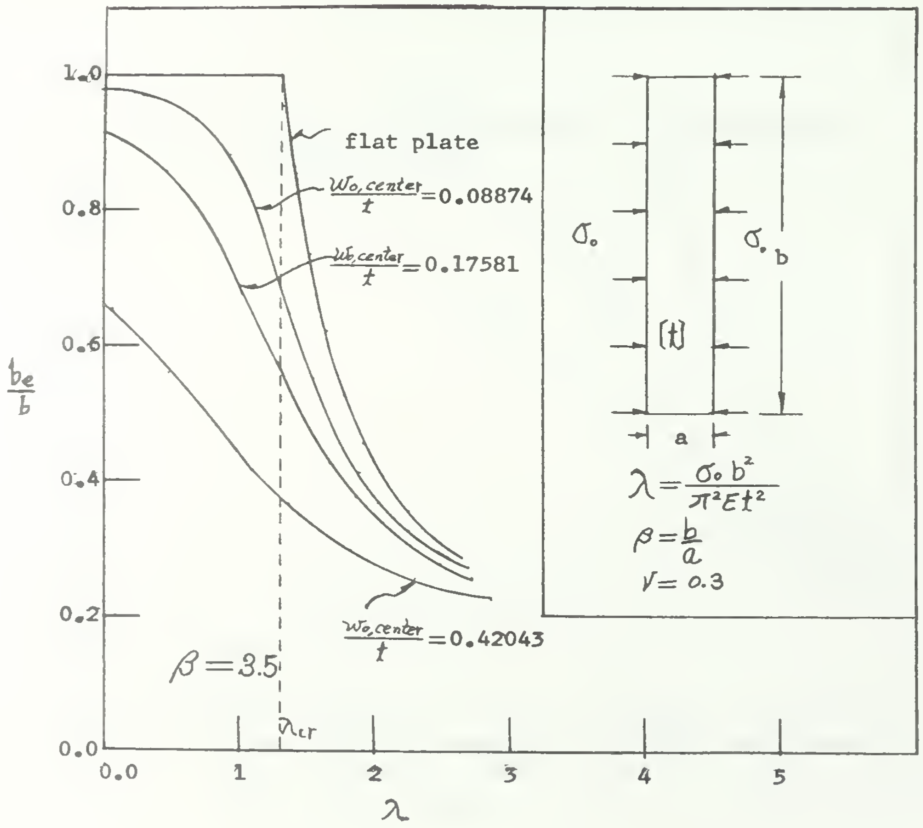

Fig. 4. EFFECTIVE WIDTH

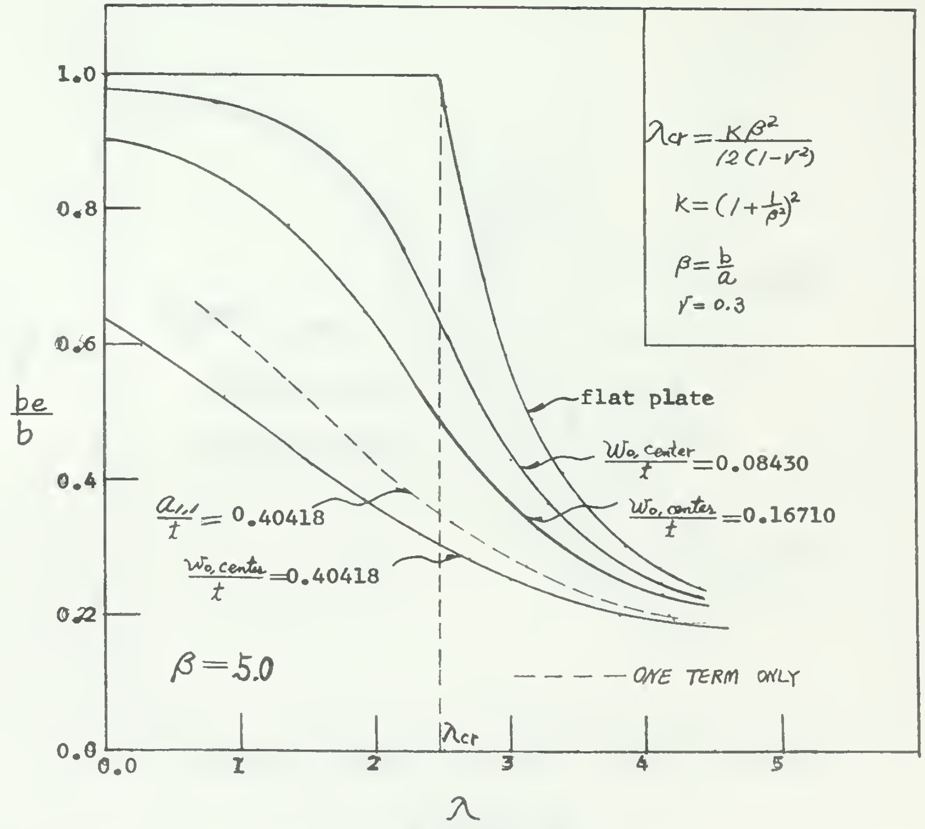

Fig. 5. EFFECTIVE WIDTH

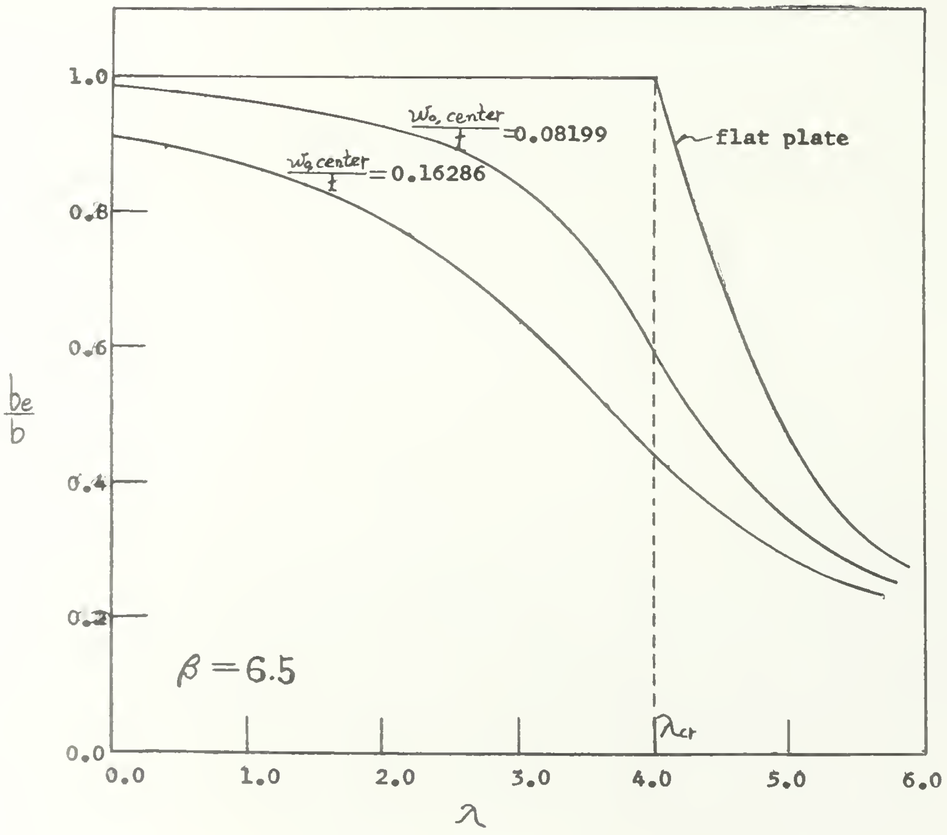

Fig. 6. EFFECTIVE WIDTH

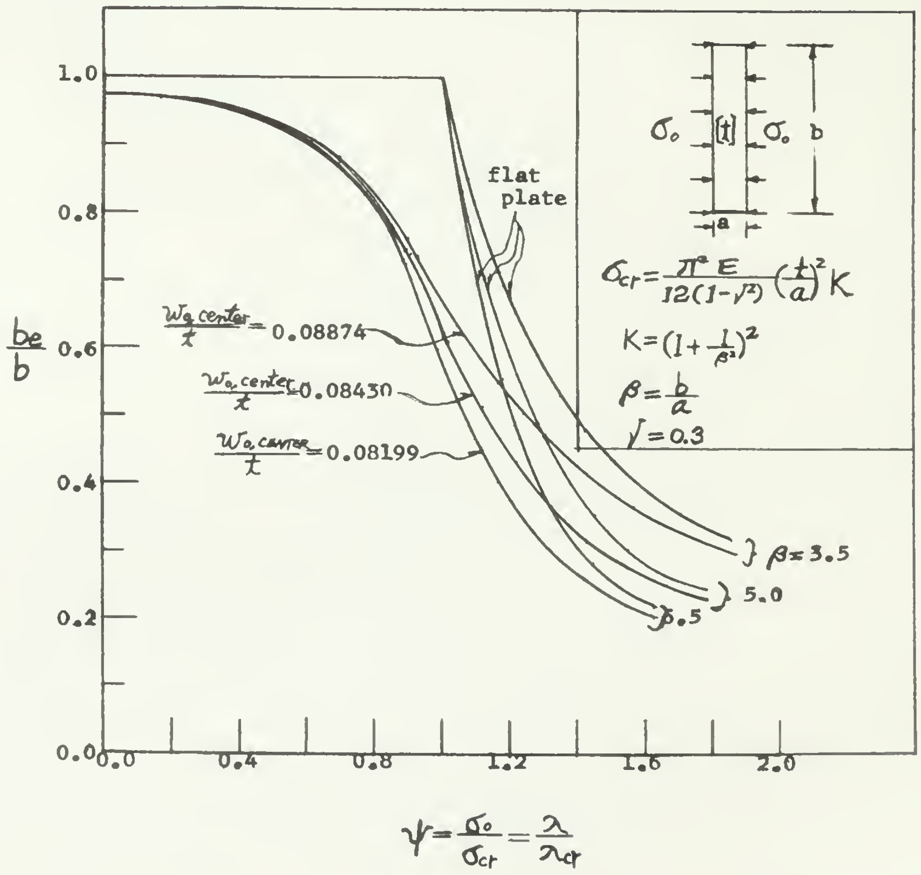

Fig. 7. COMPARISONS OF EFFECTIVE WIDTH FOR DIFFERENT ASPECT RATIOS

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Effect of small initial deflection on ef
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